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Electrodynamics of Cholesteric Liquid Crystals†

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A continuum theory is presented for the cholesteric liquid crystals subject to electromagnetic interactions. Complete field equations, jump conditions and constitutive equations are obtained. Constitutive equations include all first and second degree effects some of which are new. These include the curvature piezomagnetism, electrostrictive and magnetostrictive couples, magnetization due to motion, etc. The thermodynamic restrictions and other invariance requirements are studied. The field equations are employed to study the effect of the magnetic field on the reorientation of the molecules and transition to nematic state.

1 INTRODUCTION

In a previous paper[1] we gave a continuum theory of liquid crystals based on the micropolar continuum mechanics. The theory is constructed to be nonlinear and hydrodynamic in character at the outset. Basic to this theory is the concept of orientable points with microinertia and degrees of freedom involving translations and rotations. As discussed, this theory reduces to the director theory in the special case of incompressibility, perfect alignment and thread-like elements. As a sequence to this mechanical theory[2] we presented the electrodynamics of nematic liquid crystals. The field equations and properly invariant constitutive equations were constructed which included all second degree effects such as curvature piezoelectricity, anisotropic heat and electric conduction. While some of these new effects were noted previously and director theory was used to explain others, in most of these works attention was devoted to special situations, Refs. [3], [4].

Hydrodynamic theory of liquid crystals being basically nonlinear, a unified theory involving all first and second order phenomena cannot be constructed

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as the sum of the individual theories. Moreover by its nature anisotropy dictated by the director theory is very special in that it leads to uniaxial theories (e.g. uniaxial magnetization). For liquid crystals that cannot be approximated by straight thread-like elements that are perfectly aligned, the director theory can only represent an approximation. The theory presented in Refs. [1], and [2] and the present work employs anisotropy arising from a second order tensor so that theory is applicable to liquid crystals endowed with arbitrary inertia tensor. The state of liquid crystals theory up to 1974 is expertly presented in Refs. [3] and [4]. An examination of this and other recent literature reveal that the effect of the rate of rotation gradients are also ignored on the electromechanical behavior of liquid crystals. These effects can, not only bring their own anisotropy, but also provide other rotational dissipation and E-M effects.

The raison d'etre of the present work emanates from the above considerations. In Section 2 we present briefly a summary of the kinematics of micropolar mechanics, relevant to liquid crystals. In Section 3 the balance laws are given. These include the balance laws of micropolar continuum mechanics, the second law of thermodynamics, and Maxwell's equations expressed in a reference frame co-moving with the points of the body. Electromagnetic force and couple are those given by Dixon and Eringen,[5] De Groot and Suttorp[6] and Maugin and Eringen[7] in connection with the electrodynamics of deformable bodies. The electromagnetic fields are Laurentz invariant to within $(\mathbf{v}/\mathbf{c})^2$, where $\mathbf{v}$ is the velocity of a material point and c is the speed of light in vacuum. The jump conditions on a discontinuity surface sweeping the body are listed. In Section 4 we construct properly invariant constitutive equations for the free energy, entropy, stress, couple stress, heat conduction, polarization, magnetization and electric conduction. In Section 5 thermodynamic restrictions are studied. The theory is now complete for application. In Section 7 we indicate the passage to the director theory when the molecular elements of liquid crystals can be considered straight, thread-like filaments. In Section 8 we give the solution of the reorientation of cholesteric liquids in a channel under magnetic field. In this analysis the effect piezomagnetism is included. The cholesteric-nematic transition is discussed.

2 KINEMATICS

In our previous work[1] we gave a continuum theory of liquid crystals based on micropolar mechanics. The electromagnetic interactions with nematic liquid crystals were discussed.[2] These work conclusively indicate that the liquid crystals theory is indeed a branch of micropolar continuum theory. In a micropolar continuum a material point is considered to possess mass density

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ρ and an inertia tensor j_{kl}. The motion of a point is then equivalent to a rigid body motion. Thus a point possesses six degrees of freedom: three translations and three rotations. Referred to rectangular coordinates X_K, 1, 2, 3 in the reference state, a material point is characterized by its position vector $\mathbf{X}$ and a director Ξ attached to the point. The motion of $\mathbf{X}$ is described by the following two sets of equations

$$x_k = x_k(\mathbf{x}, t), \qquad \xi_k = \chi_{kK}(\mathbf{X}, t)\Xi_K \tag{2.1}$$

of which the first expresses the translatory motions of $\mathbf{X}$ and the second its rotations. The repeated indices are summed over 1, 2, 3. The inverse motions are posited to be unique and given by

$$X_K = X_K(\mathbf{x}, t), \qquad \Xi_K = \chi_{Kk}^{-1}\xi_k \tag{2.2}$$

where $\chi_{Kk}^{-1} = \chi_{kK}$ is an orthogonal two-point tensor so that

$$x_{k,K}X_{K,l} = \delta_{kl}, \qquad X_{K,k}x_{k,L} = \delta_{KL}$$
$$\chi_{kK}\chi_{lK} = \delta_{kl}, \qquad \chi_{kK}\chi_{kL} = \delta_{KL} \tag{2.3}$$

where δ_{kl} and δ_{KL} are Kronecker deltas.

For a fluent body, by considering the relative motions from the configurations at time t, in our work[8] (see also Ref. [1], and [9]) we introduced the following strain measures

$$c_{kl} = X_{K,k}\chi_{lK}, \qquad \gamma_{kl} = \tfrac{1}{2}\varepsilon_{kmn}\chi_{mK}\chi_{nK,l} \tag{2.4}$$

and the rate of deformation measures

$$a_{kl} = v_{l,k} + \nu_{kl}, \qquad b_{kl} = \nu_{k,l} \tag{2.5}$$

where

$$\nu_{kl} = -\varepsilon_{klm}\nu_m = \dot{\chi}_{kK}\chi_{lK} = -\nu_{lk} \tag{2.6}$$

is the so-called gyration tensor[10] and ε_{klm} is the alternating tensor. Indices following a comma denote partial differentiation with respect to space variables and a superposed dot or D/Dt indicate material derivative e.g.

$$\nu_{k,l} = \frac{\partial \nu_k}{\partial x_l}, \qquad x_{k,K} = \frac{\partial x_k}{\partial X_K}$$

$$\dot{\nu}_k \equiv \frac{D\nu_k}{Dt} = \frac{\partial \nu_k}{\partial t} + \nu_{k,l}v_l, \qquad v_k = \frac{\partial x_k(\mathbf{X}, t)}{\partial t} \tag{2.7}$$

We have shown that χ_{kK}, γ_{kl} and ν_k can be expressed in terms of an axial vector ϕ_k and its material time rate, (Ref. [8], [9]),

$$\chi_{kK} = [\cos\phi\delta_{kl} + (1 - \cos\phi)n_k n_l - \sin\phi\varepsilon_{klm}n_m]\delta_{lK}, \tag{2.8}$$

$$\gamma_{kl} = n_k\phi_{,l} + \sin\phi n_{k,l} - (1 - \cos\phi)\varepsilon_{kmn}n_m n_{n,l}, \tag{2.9}$$

$$\nu_k = \Lambda_{kl}\dot{\phi}_l \tag{2.10}$$

where

$$n_k = \frac{\phi_k}{\phi}, \qquad \phi = (\phi_k\phi_k)^{1/2},$$

$$\Lambda_{Kl} = \frac{\sin\phi}{\phi}\delta_{kl} + \left(1 - \frac{\sin\phi}{\phi}\right)n_k n_l - \frac{1}{\phi}(1 - \cos\phi)\varepsilon_{klm}n_m \tag{2.11†}$$

Here δ_{kK} is the direction cosine between the spatial and material frames. When these frames coincide δ_{kK} becomes Kronecker delta.

3 BALANCE LAWS

The balance laws of liquid crystals are the same as those of the micropolar continua. They are given by (Ref. [8] to [11]);

Mass

$$\frac{\partial\rho}{\partial t} + (\rho v_k)_{,k} = 0 \qquad \text{in } \mathscr{V} - \sigma \tag{3.1}$$

Microinertia

$$\frac{Dj_{kl}}{Dt} - \nu_{km}j_{lm} - \nu_{lm}j_{km} = 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.2}$$

Momentum

$$t_{kl,k} + \rho(f_l - \dot{v}_l) = 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.3}$$

Moment of momentum

$$m_{kl,k} + \varepsilon_{lmn}t_{mn} + \rho(l_l - \dot{\sigma}_l) = 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.4}$$

Energy

$$\rho\dot{\varepsilon} - t_{kl}a_{kl} - m_{kl}b_{lk} - q_{k,k} - \rho h = 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.5}$$

† In Ref. [1], Eq. (2.18) contains a typographical error. In the expression Λ_{kl} the coefficient of the last term, $1 - \cos\phi$, should be replaced by $(1/\phi)(1 - \cos\phi)$.

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Entropy

$$\rho\dot{\eta} - \left(\frac{q_k}{\theta}\right)_{,k} - \rho b \geq 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.6}$$

where

ρ = mass density	v_k = velocity vector
j_{kl} = microinertia tensor	ν_{kl} = gyration tensor
t_{kl} = stress tensor	f_l = body force density
m_{kl} = couple stress tensor	l_l = body couple density
ε = internal energy density	q_k = heat vector
η = entropy density	θ = absolute temperature
h = energy source	b = entropy source

Equations (3.1)–(3.5) are respectively, the local balance laws of mass, microinertia, momentum, moment of momentum and energy and Eq. (3.6) is the expression of the second law of thermodynamics. The spin inertia $\dot{\sigma}_k$ is defined by

$$\dot{\sigma}_k = \frac{D}{Dt}(j_{kl}\nu_l) = j_{kl}\dot{\nu}_l - \varepsilon_{kmr}j_{lm}\nu_r\nu_l \tag{3.7}$$

These laws are valid in the body having a material volume $\mathscr{V}$ excluding a discontinuity surface σ which may be sweeping the body with a velocity **u** in the direction of its normal. For future use we also need the integral of Eq. (3.2) as given by Eringen[12]

$$j_{kl} = J_{KL}\chi_{kK}\chi_{lL} \tag{3.8}$$

where J_{KL} is the microinertia tensor at the natural state of the body.

The mechanical balance laws are supplemented by the electromagnetic balance laws:

Guass' law

$$\nabla \cdot \mathbf{D} = q_f \quad \text{in } \mathscr{V} - \sigma \tag{3.9}$$

Faraday's law

$$\nabla \times \mathscr{E} + \frac{1}{c}\overset{*}{\mathbf{B}} = \mathbf{0} \quad \text{in } \mathscr{V} - \sigma \tag{3.10}$$

Magnetic flux

$$\nabla \cdot \mathbf{B} = 0 \quad \text{in } \mathscr{V} - \sigma \tag{3.11}$$

Ampere's law

$$\nabla \times \mathscr{H} - \frac{1}{c}\overset{*}{\mathbf{D}} - \frac{1}{c}\mathscr{J} = \mathbf{0} \quad \text{in } \mathscr{V} - \sigma \tag{3.12}$$

These laws are expressed in a coordinate frame comoving with the points of the body so that they are Laurentz invariant, to within $(\mathbf{v}/\mathbf{c})^2$, in terms of the fields defined by

$$\mathscr{E} = \mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}, \qquad \mathscr{H} = \mathbf{H} - \frac{1}{c}\mathbf{v} \times \mathbf{D},$$

$$\mathscr{J} = \mathbf{J} - q_f\mathbf{v}, \qquad \mathscr{M} = \mathbf{M} + \frac{1}{c}\mathbf{v} \times \mathbf{P} \tag{3.13}$$

where **c** is the speed of light in vacuum, and $\mathscr{E}$, $\mathscr{H}$, $\mathscr{J}$ and $\mathscr{M}$ are respectively the electric field, the magnetic field, the conduction current and the magnetization expressed in terms of the laboratory fields.

D = dielectric displacement vector
E = electric field vector
B = magnetic flux vector
H = magnetic field vector
P = polarization vector
M = magnetization vector
J = total current vector
q_f = free charge density

It is well-known that **P** and **M** defined by

$$\mathbf{P} = \mathbf{D} - \mathbf{E}, \qquad \mathbf{M} = \mathbf{B} - \mathbf{H} \tag{3.14}$$

arise from electric and magnetic multipoles. In Eqs. (3.10) and (3.12) a "star derivative" occurs. This is defined by

$$\overset{*}{A}_k = \dot{A}_k + A_k v_{l,l} - A_l v_{k,l} \tag{3.15}$$

The body force **f**, body couple **l** and the energy source h is the sum of purely mechanical parts and $E - M$ parts, i.e.

$$\rho\mathbf{f} = \rho\mathbf{f}_0 + {}_M\mathbf{f}, \qquad \rho\mathbf{l} = \rho\mathbf{l}_0 + {}_M\mathbf{l},$$
$$\rho h = \rho h_0 + {}_M h \tag{3.16}$$

where $\mathbf{f}_0$, $\mathbf{l}_0$ and $\mathbf{h}_0$ are of purely mechanical origin and ${}_M\mathbf{f}$, ${}_M\mathbf{l}$ and ${}_M h$ are the contributions arising from the interaction of E-M fields with the body.[7]

$${}_M\mathbf{f} = q_f\mathscr{E} + \frac{1}{c}(\mathscr{J} + \overset{*}{\mathbf{P}}) \times \mathbf{B} + (\mathbf{P}\cdot\nabla)\mathscr{E} + (\nabla\mathbf{B})\mathscr{M},$$

$${}_M\mathbf{l} = \mathbf{P} \times \mathscr{E} + \mathscr{M} \times \mathbf{B},$$

$${}_M h = \rho\mathscr{E}\cdot\left(\frac{\mathbf{P}}{\rho}\right)^{\cdot} - \mathscr{M}\cdot\dot{\mathbf{B}} + \mathscr{J}\cdot\mathscr{E} - {}_M\underset{\sim}{l}\cdot v \tag{3.17}$$

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For some purpose we also need electromagnetic momentum† **G** and the Pointing vector $\mathscr{S}$:

$$\mathbf{G} = \frac{1}{c}\mathbf{E} x\, \mathbf{B}, \quad \mathscr{S} = c\boldsymbol{\mathscr{E}} x\, \mathscr{H} \tag{3.18}$$

We now introduce the modified Helmholtz' free energy

$$\psi = \varepsilon - \theta\eta - \frac{1}{\rho}\mathbf{P}\cdot\boldsymbol{\mathscr{E}} \tag{3.19}$$

to replace ε in Eq. (3.5) and write the classical expression of $b = h_0/\theta$ in Eq. (3.6). Upon eliminating h_0 between Eqs. (3.5) and (3.6) we obtain the generalized *Clausius–Duhem* (C–D) *inequality* for the micropolar electromagnetic bodies:

$$-\rho(\dot{\psi} + \eta\dot{\theta}) + t_{kl}a_{kl} + m_{kl}b_{lk} + \theta^{-1}\mathbf{q}\cdot\nabla\theta - \mathbf{P}\cdot\dot{\boldsymbol{\mathscr{E}}} - \boldsymbol{\mathscr{M}}\cdot\dot{\mathbf{B}} - {}_{M}l\cdot v + \boldsymbol{\mathscr{J}}\cdot\boldsymbol{\mathscr{E}} \geq 0 \tag{3.20}$$

This inequality places restrictions on the state of the body and it is fundamental in the development of the constitutive theory.

Jump conditions On a discontinuity surface σ which may be sweeping the body with a velocity **u** in the direction of its unit normal **n** the following jump conditions must be satisfied:

$$[\rho(\mathbf{v} - \mathbf{u})]\cdot\mathbf{n} = 0, \tag{3.21}$$

$$[\rho j_{kl}(v_r - u_r)]n_r = 0, \tag{3.22}$$

$$[\rho(v_k - u_k)v_l - t_{kl} - ({}_{M}t_{kl} + u_k G_l)]n_k = 0, \tag{3.23}$$

$$[\rho\sigma_l(v_k - u_k) - m_{kl}]n_k = 0, \tag{3.24}$$

$$\begin{aligned}[(\tfrac{1}{2}\rho\mathbf{v}\cdot\mathbf{v} + \tfrac{1}{2}\rho\boldsymbol{\sigma}\cdot\mathbf{v} + \rho\varepsilon + \tfrac{1}{2}\mathbf{E}\cdot\mathbf{E} + \tfrac{1}{2}\mathbf{B}\cdot\mathbf{B})(v_k - u_k)\\ - (t_{kl} + {}_{M}t_{kl} + u_k G_l)v_l - (q_k - \mathscr{S}_k)]n_k = 0\end{aligned} \tag{3.25}$$

$$[\rho\eta(v_k - u_k) + \theta^{-1}q_k]n_k \geq 0 \tag{3.26}$$

These are the balance laws accompanying each one of the mechanical laws Eqs. (3.1)–(3.6). In these expressions we introduced the *Maxwell stress tensor* ${}_{M}t$ by

$${}_{M}t_{kl} = P_k\mathscr{E}_l - B_k\mathscr{M}_l + E_kE_l + B_kB_l - \tfrac{1}{2}(\mathbf{E}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{B} - 2\boldsymbol{\mathscr{M}}\cdot\mathbf{B})\delta_{kl} \tag{3.27}$$

† $E - M$ momentum in the comoving frame is given by $\mathscr{G} = (1/c)\boldsymbol{\mathscr{E}} x \boldsymbol{\mathscr{B}}$ where $\boldsymbol{\mathscr{B}} = \mathscr{H} + \boldsymbol{\mathscr{M}}$ Ref. [7].

The $E - M$ fields on σ are subject to the jump conditions

$$[\mathbf{D}]\cdot\mathbf{n} = w_f, \tag{3.28}$$

$$\mathbf{n} \times \left[\mathscr{E} + \frac{1}{c}\mathbf{B} \times (\mathbf{v} - \mathbf{u})\right] = 0, \tag{3.29}$$

$$[\mathbf{B}]\cdot\mathbf{n} = 0, \tag{3.30}$$

$$\mathbf{n} \times \left[\mathscr{H} - \frac{1}{c}\mathbf{D} \times (\mathbf{v} - \mathbf{u})\right] = \frac{1}{c}\mathscr{K}; \qquad \mathscr{K}\cdot\mathbf{n} = 0 \tag{3.31}$$

where w_f and $\mathscr{K}$ are respectively the surface charge density and surface current density on σ. By taking σ to coincide with the surface of the body from Eqs. (3.21)–(3.31) we obtain the boundary conditions on the surface of the body.

4 CONSTITUTIVE EQUATIONS

The state of cholesteric liquid crystals subject to electromechanical effects is determined by the characterization of the dependent variables (response functions)

$$\mathscr{Z} \equiv \{\psi, \eta, t_{kl}, m_{kl}, q_k, P_k, \mathscr{M}_k, \mathscr{J}_k\} \tag{4.1}$$

as functions of certain independent variables that characterize the constitution of the body in motion. Appropriate independent variables are

$$\mathscr{Y} \equiv \{\rho^{-1}, \theta, j_{kl}, \gamma_{kl}, a_{kl}, b_{kl}, \theta_{,k}, \mathscr{E}_k, \mathscr{B}_k\} \tag{4.2}$$

The constitutive equations are written for *each* member of Eq. (4.1) in terms of *all* members of Eq. (4.2) as

$$\mathscr{Z} = \mathscr{F}(\mathscr{Y}) \tag{4.3}$$

For ψ and η, $\mathscr{F}$ is scalar-valued; for **t** and **m** it is tensor-valued and for **q**, **P**, $\mathscr{M}$ and $\mathscr{J}$ it is vector-valued.

The constitutive equations are restricted by

i) the axiom of objectivity (material frame-indifference),
ii) the second law of thermodynamics, and
iii) invariance under time reversal.

The axiom of objectivity state that the response functions are form-invariant under time-dependent rigid motions of the spatial frame of reference, with density fixed, as described by

$$\bar{\mathbf{x}}(\mathbf{X}, t) = \mathbf{Q}(t)\mathbf{x}(\mathbf{X}, t) + \mathbf{c}_0(t), \qquad \bar{\chi}_K(\mathbf{X}, t) = \mathbf{Q}(t)\chi_K(\mathbf{X}, t) \tag{4.4}$$

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where $\mathbf{c}_0(t)$ is an arbitrary time-dependent translation and $\mathbf{Q}$ represent the *proper* group of orthogonal transformations, i.e.,

$$\mathbf{Q}\mathbf{Q}^T = \mathbf{Q}^T\mathbf{Q} = \mathbf{I}, \qquad \det \mathbf{Q} = 1 \tag{4.5}$$

Accordingly we must have:

$$\bar{\mathscr{Z}} = \mathscr{F}(\bar{\mathscr{Y}}) \tag{4.6}$$

where $\bar{\mathscr{Y}}$ and $\bar{\mathscr{Z}}$ are given by

$$\bar{\mathscr{Y}} = \{\rho^{-1}, \theta, \mathbf{QjQ}^T, \mathbf{Q}\boldsymbol{\gamma}\mathbf{Q}^T, \mathbf{QaQ}^T, \mathbf{QbQ}^T, \mathbf{Q}\nabla\boldsymbol{\theta}, \mathbf{Q}\boldsymbol{\mathscr{E}}, \mathbf{QB}\}$$

$$\bar{\mathscr{Z}} = \{\psi, \eta, \mathbf{QtQ}^T, \mathbf{QmQ}^T, \mathbf{Qq}, \mathbf{QP}, \mathbf{Q}\boldsymbol{\mathscr{M}}, \mathbf{Q}\boldsymbol{\mathscr{J}}\} \tag{4.7}$$

Except for $\mathbf{Q}$ being proper group of orthogonal transformations, all equations are the same as in our work on nematic liquid crystals treated in Ref. [2]. Employing the C–D inequality in the same way we find that

$$\psi = \psi(\rho^{-1}, \theta, \mathbf{j}, \boldsymbol{\gamma}, \boldsymbol{\mathscr{E}}, \mathbf{B}), \tag{4.8}$$

$$\eta = -\frac{\partial\psi}{\partial\theta}, \tag{4.9}$$

$$t_{kl} = {}_Et_{kl} + {}_Dt_{lk}, \tag{4.10}$$

$$m_{kl} = {}_Em_{kl} + {}_Dm_{kl}, \tag{4.11}$$

$${}_Et_{kl} = -\pi\delta_{kl} - {}_Em_{kr}\gamma_{rl}, \qquad \pi = -\frac{\partial\psi}{\partial\rho^{-1}} \tag{4.12}$$

$${}_Em_{kl} = \rho\frac{\partial\psi}{\partial\gamma_{lk}}, \tag{4.13}$$

$$P_k = -\rho\frac{\partial\psi}{\partial\mathscr{E}_k} \tag{4.14}$$

$$\mathscr{M}_k = -\rho\frac{\partial\psi}{\partial B_k} \tag{4.15}$$

and the dissipative stress ${}_D\mathbf{t}$, couples stress ${}_D\mathbf{m}$, the heat $\mathbf{q}$ and the current $\boldsymbol{\mathscr{J}}$ are subject to

$${}_Dt_{kl}a_{lk} + {}_Dm_{kl}b_{lk} + \frac{1}{\theta}q_k\theta_{,k} + \mathscr{J}_k\mathscr{E}_k \geq 0 \tag{4.16}$$

If we assume that ${}_D\mathbf{t}$, ${}_D\mathbf{m}$, $\mathbf{q}$ and $\boldsymbol{\mathscr{J}}$ are continuous functions of $\mathscr{Y}$ it follows from Eq. (4.16) that

$${}_D\mathbf{t} = {}_D\mathbf{m} = \mathbf{0}, \qquad \mathbf{q} = \boldsymbol{\mathscr{J}} = \mathbf{0} \quad \text{when} \quad \mathbf{a} = \mathbf{b} = \mathbf{0}, \nabla\theta = \boldsymbol{\mathscr{E}} = \mathbf{0} \tag{4.17}$$

For ${}_D\mathbf{t}$, ${}_D\mathbf{m}$, $\mathbf{q}$ and $\mathscr{J}$ we still have to write equations of the form Eq. (4.3) subject to Eq. (4.16) and consequently Eq. (4.17). Once the form of ψ is known the equilibrium parts of the constitutive functions η, ${}_E\mathbf{t}$, ${}_E\mathbf{m}$, $\mathbf{P}$ and $\mathscr{M}$ are fully determined by Eqs. (4.9), (4.12), (4.13)–(4.15). Finally the invariance under time reversal requires that ψ and the entropy production Eq. (4.16) do not alter their signs when the sign of time is reversed. Thus we have:

THEOREM. *Constitutive equations of cholesteric liquid crystals are given by Eqs.* (4.8)–(4.15) *subject to Eqs.* (4.16), (4.17) *and the invariance under axioms of objectivity and time reversal.*

5 POLYNOMIAL CONSTITUTIVE EQUATIONS

A Equilibrium constitutive equations

We consider that ψ is a polynomial in the variables $\mathbf{j}$, γ, $\mathscr{E}$ and $\mathbf{B}$. According to the axiom of objectivity we must have

$$\psi(\rho^{-1}, \theta, \mathbf{QjQ}^T, \mathbf{Q}\gamma\mathbf{Q}^T, \mathbf{Q}\mathscr{E}, \mathbf{QB}) = \psi(\rho^{-1}, \theta, \mathbf{j}, \gamma, \mathscr{E}, \mathbf{B}) \qquad (5.1)$$

for all members of the proper group of orthogonal transformations $\{\mathbf{Q}\}$. This means that ψ will be a scalar polynomial in the joint invariants of two symmetric tensors $\mathbf{j}$ and γ_S and three skew-symmetric tensors γ_A, $\mathscr{E}_D$ and $\mathbf{B}_D$ defined by

$$\gamma_S = \tfrac{1}{2}(\gamma + \gamma^T), \qquad \gamma_A = \tfrac{1}{2}(\gamma - \gamma^T),$$
$$\mathscr{E}_{Dkl} = \varepsilon_{klm}\mathscr{E}_m, \qquad B_{Dkl} = \varepsilon_{klm}B_m \qquad (5.2)$$

Although the complete set of invariants of these tensors can be read from a table given by Smith,[13] this list is rather long. Therefore we give below only the invariants whose total degree does not exceed three. In this way our constitutive equations for ${}_E\mathbf{m}$, $\mathbf{P}$ and $\mathscr{M}$ will be of degree two in these matrices. The invariants whose degree is higher than the third contribute little and in nearly all applications are unimportant. With the inclusion of the third degree invariants almost all known physical phenomena are accounted for.

$$\begin{aligned}
&I_1 = \operatorname{tr}\gamma, \quad I_2 = \operatorname{tr}\gamma^2, \quad I_3 = \operatorname{tr}(\gamma\gamma^T), \quad I_4 = \operatorname{tr}\mathbf{j},\\
&I_5 = \operatorname{tr}(\gamma\mathbf{j}), \quad I_6 = \operatorname{tr}(\gamma^2\mathbf{j}), \quad I_7 = \operatorname{tr}(\gamma^T\gamma\mathbf{j}), \quad I_8 = \operatorname{tr}(\gamma\gamma^T\mathbf{j}),\\
&I_9 = \mathscr{E}\cdot\mathscr{E}, \quad I_{10} = \mathscr{E}\cdot\mathbf{j}\mathscr{E}, \quad I_{11} = \mathscr{E}\cdot\gamma\mathscr{E}, \quad I_{12} = \operatorname{tr}(\gamma\mathscr{E}_D),\\
&I_{13} = \operatorname{tr}(\mathbf{j}\gamma\mathscr{E}_D), \quad I_{14} = \operatorname{tr}(\mathbf{j}\gamma^T\mathscr{E}_D), \quad I_{15} = \operatorname{tr}(\gamma^3\mathscr{E}_D),\\
&I_{16} = \mathbf{B}\cdot\mathbf{B}, \quad I_{17} = \mathbf{B}\cdot\mathbf{jB}, \quad I_{18} = \mathbf{B}\cdot\gamma\mathbf{B}, \quad I_{19} = \operatorname{tr}\gamma_S^3,\\
&I_{20} = \operatorname{tr}(\gamma_S\gamma_A^2)
\end{aligned} \qquad (5.3)$$

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From this list several invariants involving the first power of $\mathbf{B}$ are excluded since they violate the axiom of time reversal. This is because upon the change of the sign of time $\mathbf{B}$ reverses its sign but $\mathscr{E}$ does not. In addition we assumed that the natural state ($\gamma = \mathbf{0}, \mathscr{E} = \mathbf{0}, \mathbf{B} = \mathbf{0}$) is stress-free.

The polynomial form of ψ, up to and including all third degree terms, is therefore given by

$$\begin{aligned}\psi = {} & \tfrac{1}{2}A_1 I_1^2 + \tfrac{1}{2}A_3 I_1^2 I_4 + A_3 I_1 I_5 + \tfrac{1}{2}A_4 I_2 + \tfrac{1}{2}A_5 I_2 I_4 + \tfrac{1}{2}A_6 I_3 \\ & + \tfrac{1}{2}A_7 I_3 I_4 + A_8 I_6 + \tfrac{1}{2}A_9 I_7 + \tfrac{1}{2}A_{10} I_8 + \bar{A}_1 I_1 I_2 + \bar{A}_2 I_1 I_3 \\ & + \bar{A}_3 I_1^3 + \bar{A}_4 I_{19} + \bar{A}_5 I_{20} - \frac{1}{2\rho}[(e_1 + e_2 I_4 + \bar{e}_2 I_1) I_9 \\ & + e_3 I_{10} + \bar{e}_3 I_{11} + 2(e_4 + e_5 I_4 + \bar{e}_5 I_1) I_{12} + 2e_6 I_{13} \\ & + 2e_7 I_{14} + 2\bar{e}_6 I_{15} + (\mu_1 + \mu_2 I_4 + \bar{\mu}_2 I_1) I_{16} + \mu_3 I_{17} + \bar{\mu}_3 I_{18}] \end{aligned} \tag{5.4}$$

where $A_i, \bar{A}_i, e_i, \bar{e}_i, \mu_i$ and $\bar{\mu}_i$ are functions of ρ^{-1} and θ only. Carrying ψ into Eqs. (4.13)–(4.15) we obtain:

$$\begin{aligned}\frac{{}_E\mathbf{m}^T}{\rho} = {} & \Big[A_1 \operatorname{tr}\gamma + A_2 \operatorname{tr}\mathbf{j}\operatorname{tr}\gamma + A_3 \operatorname{tr}(\gamma\mathbf{j}) + \bar{A}_1 \operatorname{tr}\gamma^2 \\ & + \bar{A}_2 \operatorname{tr}(\gamma\gamma^T) + 3\bar{A}_3(\operatorname{tr}\gamma)^2 - \frac{1}{2\rho}(\bar{e}_2\mathscr{E}^2 + \bar{\mu}_2\mathbf{B}^2 + 2\bar{e}_5 \operatorname{tr}\gamma\mathscr{E}_D)\Big]\mathbf{I} \\ & + A_3\mathbf{j}\operatorname{tr}\gamma + (A_4 + A_5\operatorname{tr}\mathbf{j} + 2\bar{A}_1\operatorname{tr}\gamma)\gamma^T \\ & + (A_6 + A_7\operatorname{tr}\mathbf{j} + 2\bar{A}_2\operatorname{tr}\gamma)\gamma + A_8(\mathbf{j}\gamma^T + \gamma^T\mathbf{j}) + A_9\gamma\mathbf{j} \\ & + A_{10}\mathbf{j}\gamma + 3\bar{A}_4\gamma_S^2 + \bar{A}_5(\gamma_A^2 - \gamma_S\gamma_A - \gamma_A\gamma_S) \\ & - \frac{1}{2\rho}[\bar{e}_3\mathscr{E}\otimes\mathscr{E} - 2(e_4 + e_5\operatorname{tr}\mathbf{j} + \bar{e}_5\operatorname{tr}\gamma)\mathscr{E}_D \\ & - 2e_6\mathbf{j}\mathscr{E}_D + 2e_7\mathscr{E}_D\mathbf{j} - 2\bar{e}_6(\gamma^T\mathscr{E}_D + \mathscr{E}_D\gamma^T) + \bar{\mu}_3\mathbf{B}\otimes\mathbf{B}], \end{aligned} \tag{5.5}$$

$$\begin{aligned}\mathbf{P} = {} & [(e_1 + e_2\operatorname{tr}\mathbf{j} + \bar{e}_2\operatorname{tr}\gamma)\mathbf{I} + e_3\mathbf{j} + \bar{e}_3\gamma_S]\mathscr{E} - (e_4 + e_5\operatorname{tr}\mathbf{j} \\ & + \bar{e}_5\operatorname{tr}\gamma)\gamma_D - e_6(\mathbf{j}\gamma)_D + e_7(\gamma\mathbf{j})_D - \bar{e}_6(\gamma^2)_D, \end{aligned} \tag{5.6}$$

$$\mathscr{M} = [(\mu_1 + \mu_2\operatorname{tr}\mathbf{j} + \bar{\mu}_2\operatorname{tr}\gamma)\mathbf{I} + \mu_3\mathbf{j} + \bar{\mu}_3\gamma_S]\mathbf{B} \tag{5.7}$$

where a subscript D attached to tensors and vectors indicate their dual defined by

$$T_{Di} = \varepsilon_{ijk}T_{jk}, \qquad \mathscr{E}_{Dij} = \varepsilon_{ijk}\mathscr{E}_k \tag{5.8}$$

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Several observations are in order:

a) Constitutive Eqs. (5.4)–(5.6) differ from the corresponding ones for the nematic liquid crystals,[2] in the terms whose coefficients carry a superposed bar. These terms are admissible here because of the fact that the symmetry group is the proper group of orthogonal transformation and does not contain reflection. As a result additional physical effects appear that are not admissible for the nematic liquids. For example we have the *electrostrictive* and *magnetostrictive couple stresses* represented by the terms with coefficients $(\bar{e}_2, \bar{e}_3)$ and $(\bar{\mu}_2, \bar{\mu}_3)$ respectively in Eq. (5.5). The terms with coefficients $\bar{e}_5$ and $\bar{e}_6$ are also couple stresses arising from the interaction of the electric field with curvature.

b) "The curvature piezoelectricity" represented by the coefficients e_4 to e_7 are identical to those encountered in the theory of nematic liquid crystals. This is as expected. These effects were studied.[14] In the polarization we notice however additional effects (the terms containing $\bar{e}_5$ and $\bar{e}_6$ in Eq. (5.6)) which do not change sign with the reversal of the sign of γ. We may call these second order effects *curvature polarization*. The director forms of the invariants $I_1 I_{12}$ and I_{15} responsible for these effects are obtained via the correspondance

$$\chi_{k3} = n_k, \qquad \mathbf{n} \cdot \mathbf{n} = 1, \qquad j_{kl} = I_0(\delta_{kl} - n_k n_l) \tag{5.9}$$

established in Ref. [2]. Hence

$$\begin{aligned} I_1 I_{12} &= \tfrac{1}{2}\mathbf{n} \cdot (\nabla \times \mathbf{n})[\mathcal{E} \cdot (\mathbf{n} \cdot \nabla)\mathbf{n} - \mathbf{n} \cdot \mathcal{E}\nabla \cdot \mathbf{n}], \\ I_{15} &= \tfrac{1}{4}\varepsilon_{lpq} n_p n_{q,i} n_{j,l}(\mathcal{E} \cdot \mathbf{n}\delta_{ij} - n_i \mathcal{E}_j). \end{aligned} \tag{5.10}$$

From these it is clear that upon the inversion $\mathbf{x} \to -\mathbf{x}$ these quantities do not change sign. Thus the polarization produced would not alter its sign with sign of curvature or twist. Note that the term containing $\bar{e}_6$ is also of the same nature however being purely the result of the orientation gradients.

c) Magnetization Eq. (5.7) is also affected by the curvature. The terms containing $\bar{\mu}_2$ and $\bar{\mu}_3$ which are responsible for the magnetostrictive couple stress, have no counterpart in classical electromagnetism. The director limit of the invariant I_{18} responsible for this effect is obtained to be

$$I_{18} = \tfrac{1}{2}(\mathbf{B} \times \mathbf{n}) \cdot (\mathbf{B} \cdot \nabla)\mathbf{n} \tag{5.11}$$

Hence magnetization will result upon the interaction of the rotation gradients with the magnetic field in the form

$$\tfrac{1}{4}\bar{\mu}_3[\mathbf{n} \times (\mathbf{B} \cdot \nabla)\mathbf{n} + (\nabla\mathbf{n}) \cdot (\mathbf{B} \times \mathbf{n})] \tag{5.12}$$

It is expected that this second order effect is also very small.

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B Non-equilibrium constitutive equations

To construct the polynomial constitutive equations for the nonequilibrium fields ${}_D\mathbf{t}$, ${}_D\mathbf{m}$, $\mathbf{q}$ and $\mathscr{J}$ we need to list the generators of these quantities in terms of the argument tensors and vectors $\boldsymbol{\gamma}$, $\mathbf{j}$, $\mathbf{a}$, $\mathbf{b}$, $\nabla\theta$, $\mathscr{E}$ and $\mathbf{B}$. The number of these generators are too many for the exact constitutive equations. The generators having high degrees in the products of these vectors and tensors lead to nonlinear terms which are not important from the point of view of applications. We therefore construct only constitutive equations that are linear in these fields, and their products with $\mathbf{j}$ since $\mathbf{j}$ is the anisotropy indicator. Moreover we exclude $\boldsymbol{\gamma}$ from our list since the viscoelastic effects in torsion (hence the relaxation of elastic twist) is not considered here. For simplicity we also neglect the effect of $\mathbf{b}$. This implies that viscous forces arising from rotational gradients are negligible. Consequently C–D inequality Eq. (4.16) gives.†

$$ {}_D\mathbf{m} = \mathbf{0} \tag{5.13}$$

Note that viscous forces due to intrinsic rotations are included. With these considerations we obtain:

$$\begin{aligned} {}_D\mathbf{t} = {} & (\alpha_1 \operatorname{tr} \mathbf{a} + \alpha_2 \operatorname{tr} \mathbf{a} \operatorname{tr} \mathbf{j} + \alpha_2 \operatorname{tr} \mathbf{ja})\mathbf{I} + \alpha_4 \mathbf{j} \operatorname{tr} \mathbf{a} \\ & + (\alpha_5 + \alpha_6 \operatorname{tr} \mathbf{j})\mathbf{a} + (\alpha_7 + \alpha_8 \operatorname{tr} \mathbf{j})\mathbf{a}^T + \alpha_9 \mathbf{ja} + \alpha_{10}\mathbf{aj} \\ & + \alpha_{11}\mathbf{ja}^T + \alpha_{12}\mathbf{a}^T\mathbf{j} + [(\alpha_{13} + \alpha_{14} \operatorname{tr} \mathbf{j})\mathbf{I} + \alpha_{15}\mathbf{j}](\nabla\theta)_D \\ & + \alpha_{16}(\nabla\theta)_D\mathbf{j} + [(\alpha_{17} + \alpha_{18} \operatorname{tr} \mathbf{j})\mathbf{I} + \alpha_{19}\mathbf{j}]\mathscr{E}_D + \alpha_{20}\mathscr{E}_D\mathbf{j}, \end{aligned} \tag{5.14}$$

$$\begin{aligned} \frac{\mathbf{q}}{\theta} = {} & [(\kappa_1 + \kappa_2 \operatorname{tr} \mathbf{j})\mathbf{I} + \kappa_3\mathbf{j}]\nabla\theta + [\kappa_4 + \kappa_5 \operatorname{tr} \mathbf{j})\mathbf{I} + \kappa_6\mathbf{j}]\mathscr{E} \\ & + (\kappa_7 + \kappa_8 \operatorname{tr} \mathbf{j})\mathbf{a}_D + \kappa_9(\mathbf{aj})_D + \kappa_{10}(\mathbf{ja})_D, \end{aligned} \tag{5.15}$$

$$\begin{aligned} \mathscr{J} = {} & [(\sigma_1 + \sigma_2 \operatorname{tr} \mathbf{j})\mathbf{I} + \sigma_3\mathbf{j}]\mathscr{E} + [(\sigma_4 + \sigma_5 \operatorname{tr} \mathbf{j})\mathbf{I} + \sigma_6\mathbf{j}]\nabla\theta \\ & + (\sigma_7 + \sigma_8 \operatorname{tr} \mathbf{j})\mathbf{a}_D + \sigma_9(\mathbf{aj})_D + \sigma_{10}(\mathbf{ja})_D \end{aligned} \tag{5.16}$$

where α_i, κ_i and σ_i are functions of ρ^{-1}, θ. From these equations we have excluded several terms involving $\mathbf{B}$ in anticipating that the second law of thermodynamics would annihilate these terms.

According to the axiom of time reversal, the entropy production Eq. (4.16) must not alter its form when the sign of time is reversed. The implication of this invariance is well-known in classical electromagnetism and studied by several authors, more recently.[15] Upon the time reversal $\mathbf{j}$, $\mathscr{E}$ and $\nabla\theta$ do not

† There is no particular difficulty in incorporating the effect of $\mathbf{b}$. In fact all we need is to repeat the terms containing $\mathbf{a}$ in these equations with $\mathbf{a}$ replaced by $\mathbf{b}$ and the coefficients α_i replaced by β_i.

alter their signs but the sign of **a** is altered. Thus, if Eq. (4.16) is to remain invariant under the time reversal, we must have

$$\kappa_7 = \alpha_{13}, \quad \kappa_8 = \alpha_{14}, \quad \kappa_9 = \alpha_{15}, \quad \kappa_{10} = \alpha_{16}$$
$$\sigma_7 = \alpha_{17}, \quad \sigma_8 = \alpha_{18}, \quad \sigma_9 = \alpha_{19}, \quad \sigma_{10} = \alpha_{20}. \tag{5.17}$$

The axiom of time reversal also implies the Onsager relations so that

$$\alpha_3 = \alpha_4, \quad \alpha_9 = \alpha_{10} \tag{5.18}$$

Several observations are in order:

a) Excluding the terms that contain $\mathscr{E}$ and $\nabla\theta$, Eq. (5.14) has the same form as that obtained for the Nematic liquid crystals.[2] The two terms containing $\mathscr{E}$ in Eq. (5.14) indicates the possibility that the electric field, irrespective of motion, can cause dynamical stress. Similarly, the terms involving $\nabla\theta$ denote the stress caused by the temperature gradient. Since both sets of terms involving $\mathscr{E}$ and $\nabla\theta$ do not contribute to the entropy production, they are hidden effects.

b) The *Peltier effect* described by the term involving $\mathscr{E}$ in Eq. (5.15) indicates that heat may be generated by the electric field. The dependence of $\mathscr{J}$ on $\nabla\theta$ in Eq. (5.16), shows that a current will flow due to temperature gradient. This is the *Seebeck effect*. Both of these effects are identical to those found in nematic liquid crystals.[2] The Peltier and Seebeck effects are well-known in classical electromagnetism. They are however, different from the present expressions in that for liquid crystals they are anisotropic as indicated by the dependence on the micro-intertia tensor **j**, hence the orientations of micro-elements.

c) The terms containing (κ_7 to κ_{10}) in Eq. (5.15) indicate the possibility of heat conduction without temperature gradient and electric field. The dynamical stress due to temperature gradient (coefficients of α_{13} to α_{16}) is consistent with an observation of Lehman reported.[16] A consequence of these terms is the production of rotation of the local molecular axes under thermal gradient parallel to the helical axis (The Lehman effect). These effects are hidden in that they do not contribute to the entropy production.

d) The terms containing α_{17} to α_{20} in Eq. (5.14) usggest that the electric field will exert stress on the cholesteric liquids. Correspondingly, the deformation rate will produce current, irrespective of the electric field and the temperature gradient, indicated by the terms containing σ_7 to σ_{10}. These are also hidden effects.

e) Finally we remark that to the same degree of approximation of non-equilibrium constitutive equations, the equilibrium constitutive Eqs.

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(5.5)–(5.7) would loose the terms containing the constitutive moduli containing $\bar{A}_i$, $\bar{e}_i$ and $\bar{\mu}_i$.

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6 THERMODYNAMIC RESTRICTION

The second law of thermodynamics Eq. (4.16) places restrictions on the non-equilibrium constitutive Eqs. (5.14)–(5.16). To determine the consequence of this inequality we first write these equations in the compact forms.

$$D^t ij = \alpha_{jikl} a_{kl} + \alpha_{jik} \theta_{,k} + e_{jik} \mathscr{E}_k, \tag{6.1}$$

$$\frac{q_i}{\theta} = \kappa_{ij} \theta_{,j} + \kappa^e_{ij} \mathscr{E}_j - \alpha_{jki} a_{jk}, \tag{6.2}$$

$$\mathscr{J}_i = \sigma_{ij} \mathscr{E}_j + \sigma^\theta_{ij} \theta_{,j} - e_{jki} a_{jk} \tag{6.3}$$

where

$$\begin{aligned}
\alpha_{ijkl} &= (\alpha_1 \delta_{kl} + \alpha_2 \operatorname{tr} \mathbf{j} \delta_{kl} + \alpha_3 j_{kl}) \delta_{ij} + \alpha_4 j_{ij} \delta_{kl} \\
&\quad + (\alpha_5 + \alpha_6 \operatorname{tr} \mathbf{j}) \delta_{il} \delta_{jk} + (\alpha_7 + \alpha_8 \operatorname{tr} \mathbf{j}) \delta_{ik} \delta_{jl} \\
&\quad + \alpha_9 \delta_{il} j_{jk} + \alpha_{10} j_{il} \delta_{jk} + \alpha_{11} \delta_{ik} \delta_{jl} + \alpha_{12} j_{ik} \delta_{jl}, \\
\alpha_{ijk} &= [(\alpha_{13} + \alpha_{14} \operatorname{tr} \mathbf{j}) \delta_{jl} + \alpha_{15} j_{jl}] \varepsilon_{ikl} + \alpha_{16} j_{il} \varepsilon_{jlk}, \\
e_{ijk} &= [(\alpha_{17} + \alpha_{18} \operatorname{tr} \mathbf{j}) \delta_{jl} + \alpha_{19} j_{jl}] \varepsilon_{ikl} + \alpha_{20} j_{il} \varepsilon_{jlk}, \\
\kappa_{ij} &= (\kappa_1 + \kappa_2 \operatorname{tr} \mathbf{j}) \delta_{ij} + \kappa_3 j_{ij}, \\
\sigma_{ij} &= (\sigma_1 + \sigma_2 \operatorname{tr} \mathbf{j}) \delta_{ij} + \sigma_3 j_{ij}, \\
\kappa^e_{ij} &= (\kappa_4 + \kappa_5 \operatorname{tr} \mathbf{j}) \delta_{ij} + \kappa_6 j_{ij}, \\
\sigma^\theta_{ij} &= (\sigma_4 + \sigma_5 \operatorname{tr} \mathbf{j}) \delta_{ij} + \sigma_6 j_{ij}
\end{aligned} \tag{6.4}$$

Substituting Eq. (6.1)–(6.3) into Eq. (4.16), we find that the contributions **a** to the entropy inequality is uncoupled from those of $\mathscr{E}$ and $\nabla\theta$. Thus Eq. (4.16) will not be violated if and only if

$$\alpha_{ijkl} a_{ij} a_{kl} \geq 0, \tag{6.5}$$

$$\kappa_{ij} \theta_{,i} \theta_{,j} + \sigma_{ij} \mathscr{E}_i \mathscr{E}_j + (\kappa^e_{ji} + \sigma^\theta_{ij}) \mathscr{E}_i \theta_{,j} \geq 0. \tag{6.6}$$

The inequality Eq. (6.5) was already investigated.[1] The necessary and sufficient conditions for Eq. (6.5) not to be violated are given by Eq. (7.12) of Ref. [1].†

† We note however, that the inequalities Eq. (7.13) of Ref. [1] derived from Eq. (7.12) with extra conditions that Eq. (7.12) remain valid for all j_{ii} are sufficient, but not necessary.

The inequality Eq. (6.6) may be written in a compact form

$$K_{\alpha\beta}\lambda_\alpha\lambda_\beta \geq 0 \qquad \alpha, \beta = 1, 2, \ldots, 6 \tag{6.7}$$

where $K_{\alpha\beta} = K_{\beta\alpha}$ and

$$\lambda_i = \theta_{,i}, \qquad \lambda_{i+3} = \mathscr{E}_i \tag{6.8}$$

$$K_{ij} = \kappa_{ij}, \qquad K_{i+3,j+3} = \sigma_{ij}$$
$$K_{i+3,j} = K_{j,i+3} = \tfrac{1}{2}(\kappa^e_{ji} + \sigma^\theta_{ij}) \qquad i, j = 1, 2, 3. \tag{6.8}$$

It is clear that $K_{\alpha\beta}$ is a symmetric, non-negative, 6×6 matrix. The necessary and sufficient conditions for Eq. (6.7) to be non-negative for all λ_α is that all eigenvalues of $K_{\alpha\beta}$ be non-negative. Alternatively, this is secured by having the sequence of subdeterminants lying on the main diagonal of the matrix $K_{\alpha\beta}$ be non-negative, i.e.,

$$\begin{aligned} \det K_{\alpha\beta} \geq 0 \quad & \text{for} \quad \alpha, \beta = 1 \\ & \text{for} \quad \alpha, \beta = 1, 2 \\ & \text{for} \quad \alpha, \beta = 1, 2, 3 \\ & \quad \vdots \\ & \text{for} \quad \alpha, \beta = 1, 2, \ldots, 6 \end{aligned} \tag{6.9}$$

The first three subdeterminants are uncoupled from the rest. It therefore follows that κ_{ij} must be non-negative matrix by itself. If we exchange $\theta_{,i}$ and $\mathscr{E}_i$ the same reasoning shows that σ_{ij} must be non-negative matrix also, i.e.

$$\kappa_{ij}\theta_{,i}\theta_{,j} \geq 0, \qquad \sigma_{ij}\mathscr{E}_i\mathscr{E}_j \geq 0. \tag{6.10}$$

Of course, the second of this is included in the conditions Eq. (6.9).

Referred to the principal axes of j_{ij}, κ_{ij} is simplified to

$$\kappa_{ij} = (\kappa_1 + \kappa_2 \operatorname{tr} \mathbf{j} + \kappa_3 j_{\underline{ii}})\delta_{ij}. \tag{6.11}$$

Therefore, κ_{ij} is a non-negative form if and only if

$$\kappa_1 + \kappa_2 \operatorname{tr} \mathbf{j} + \kappa_3 j_{\underline{ii}} \geq 0. \tag{6.12}$$

Since $j_{\underline{ii}} \geq 0$, Eq. (6.12) is not violated for all $j_{\underline{ii}}$ if and only if

$$\kappa_1 \geq 0, \qquad \kappa_2 \geq 0, \qquad 2\kappa_2 + \kappa_3 \geq 0. \tag{6.13}$$

The situation for σ_{ij} is identical to Eqs. (6.11)–(6.13). The conditions Eq. (6.9) must further be investigated for $\kappa^e_{ij} + \sigma^\theta_{ij}$. This is a routine algebra involving the expansion of the last three determinants of Eq. (6.9) which incidentally contain the conditions on σ_{ij}.

The condition on the material stability requires that the free energy must also be non-negative for all γ, $\mathscr{E}$ and $\mathscr{B}$. In the case $\bar{A}_i = \bar{e}_i = \bar{\mu}_i = 0$, the free energy is the same for the nematic and cholesteric liquid crystals, and the

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conditions are studied in Ref. [2]. The case of non-vanishing $\bar{A}_i$, $\bar{e}_i$ and $\bar{\mu}_i$ is a little more elaborate but the method of approach is similar to the one discussed in Ref. [2].

7 PASSAGE TO DIRECTOR THEORY

If the elements of liquid crystals can be considered to be thread-like. The director concept may be introduced to simplify the basic equations. Let Ξ_3 be the common orientations of the thread-like molecules at the reference state, then we may write

$$d_k = \xi_k = \chi_{kk}\Xi_k, \qquad \Xi_k = \chi_{kk}\, d_k \tag{7.1}$$

where d_k is the *director* having unit magnitude, i.e.

$$d_k d_k = 1. \tag{7.2}$$

The independent variables appearing in the constitutive equations and $\dot{\sigma}$ now take the forms

$$\begin{aligned} \sigma_k &= D(j_{kl}v_l)/D^k & \mathbf{v} &= \mathbf{d} \times \mathring{\mathbf{d}} + \tfrac{1}{2}\nabla \times \mathbf{v} \\ a_{kl} &= d_{kl} + \mathring{d}_k d_l - d_k \mathring{d}_l, & b_{kl} &= v_{k,l}, \\ \gamma_{kl} &= \tfrac{1}{2}\varepsilon_{kmn}\, d_m\, d_{n,l}, & j_{kl} &= I_0(\delta_{kl} - d_k\, d_l) \end{aligned} \tag{7.3}$$

where I_0 is the common constant value of the micro-inertia tensor about Ξ_1 and Ξ_2 axes. The invariant time rate $\mathring{\mathbf{d}}, d_{kl}$ and w_{kl} are defined by

$$\mathring{d}_k = \dot{d}_k - w_{kl}, \qquad d_{kl} = \tfrac{1}{2}(v_{k,l} + v_{l,k}), \qquad w_{kl} = \tfrac{1}{2}(v_{k,l} - v_{l,k}) \tag{7.4}$$

Employing Eqs. (7.2)–(7.4) in the balance laws Eqs. (3.1)–(3.6) and constitutive Eqs. (5.4)–(5.7) and (5.14)–(5.16), we obtain the field equations of cholesteric liquid crystals whose elements are straight thread-like elements. These equations are then solved to determine the velocity field v_k and the director field d_k. Since these manipulations are straightforward and routine, we do not reproduce the final forms of these equations here.

8 CHOLESTERIC-NEMATIC TRANSITION UNDER MAGNETIC FIELD

As an application of the theory developed, here we discuss the effect of a constant magnetic field **H** perpendicular to the twist axis of cholesteric liquid crystals. This problem has been discussed,[4,17] by use of the energy method.

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Here we employ the full constitutive Eqs. (5.5)–(5.7) so that curvature magnetostrictive effects are also taken into account. These effects appear to have been ignored in the literature. Our main purpose however is to illustrate how the present theory can be employed in a systematic way for other more complicated problems in which not only the geometry and orientations may be more complicated but also other physical effects are present (e.g. electric field, motion etc.).

We consider cholesteric liquid placed between two parallel plates with their twist axes perpendicular to the plates. A constant magnetic field **H** is applied parallel to y-axis (perpendicular to the twist axes of cholesterics). The problem is to determine the angle of rotations of cholesteric elements, as a function of **H**. The motion is being absent we have $\mathbf{a} = \mathbf{0}$. All variables are functions of z only so that

$$\phi_1 = \phi_2 = 0, \qquad \phi_3 = \phi(z) \tag{8.1}$$

From Eq. (2.11) we have $n_1 = n_2 = 0, n_3 = 1$ so that Eq. (2.9) gives

$$\gamma_{kl} = \phi'\delta_{k3}\delta_{l3} \tag{8.2}$$

From Eqs. (5.5) and (4.12) we obtain

$$_Et_{kl} = -p\delta_{kl} - \rho(A^0_{k3} + A^1_{k3}\phi' + A^2_{k3}\phi'^2)\phi'\delta_{l3}, \tag{8.3}$$

$$_Em_{kl} = \rho(A^0_{kl} + A^1_{kl}\phi' + A^2_{kl}\phi'^2), \tag{8.4}$$

where p is the pressure and

$$\begin{aligned} A^0_{kl} &= -\frac{1}{2\rho}(\bar{\mu}_2 B^2\delta_{kl} + \bar{\mu}_3 B_k B_l), \\ A^1_{kl} &= (A_1 + A_2 \operatorname{tr}\mathbf{j} + A_3 j_{33})\delta_{kl} + A_3 j_{kl} \\ &\quad + (A_4 + A_5 \operatorname{tr}\mathbf{j} + A_6 + A_7 \operatorname{tr}\mathbf{j})\delta_{k3}\delta_{l3} \\ &\quad + (A_8 + A_9)j_{k3}\delta_{l3} + (A_8 + A_{10})j_{l3}\delta_{k3}, \\ A^2_{kl} &= [(\bar{A}_1 + \bar{A}_2 + 3\bar{A}_3)\delta_{kl} + (2\bar{A}_1 + 2\bar{A}_2 + 3\bar{A}_4)\delta_{k3}\delta_{l3}] \end{aligned} \tag{8.5}$$

For the microinertia tensor by substituting Eq. (2.8) into Eq. (3.8) we will have

$$j_{kl} = G_{klmn}J^0_{mn}, \tag{8.6}$$

where

$$\begin{aligned} G_{klmn} &= \cos^2\phi\,\delta_{km}\delta_{ln} + (\cos\phi - \cos^2\phi)(n_k n_m \delta_{ln} + n_l n_n \delta_{km}) \\ &\quad + (1 - \cos\phi)^2 n_k n_l n_m n_n - \sin\phi\cos\phi(\varepsilon_{kmr}\delta_{ln}n_r + \varepsilon_{lnr}\delta_{km}n_r) \\ &\quad - \sin\phi(1 - \cos\phi)(\varepsilon_{kmr}n_l n_n n_r + \varepsilon_{lnr}n_k n_m n_r) + \sin^2\phi\,\varepsilon_{kmp}\varepsilon_{lnr}n_p n_r, \end{aligned} \tag{8.7}$$

$$J^0_{mn} = J_{MN}\delta_{mM}\delta_{nN} \tag{8.8}$$

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valid for the general case. For rod-like elements selecting X-axis along the axes of an element and θ_0 as the angle between x and X, we have for J_{KL}

$$J_{11} = 0, \qquad J_{22} = J_{33} = J^0, \quad \text{all other } J_{KL} = 0 \tag{8.9}$$

This through Eq. (8.8) gives

$$J_{11}^0 = \tfrac{1}{2}J^0\left[1 - \cos 2\left(\theta_0 + \frac{\pi z}{p}\right)\right],$$

$$J_{22}^0 = \tfrac{1}{2}J^0\left[1 + \cos 2\left(\theta_0 + \frac{\pi z}{p}\right)\right],$$

$$J_{12}^0 = \tfrac{1}{2}J^0 \sin 2\left(\theta_0 + \frac{\pi z}{p}\right), \qquad J_{13}^0 = J_{23}^0 = 0 \tag{8.10}$$

where p is the half pitch and θ_0 is the angle of the axis of elements at $z = 0$ with x-axis. Note that since rods possess no positive or negative sense the period is only p.

Employing Eq. (8.10) and $\phi_k = \phi\delta_{k3}$, $n_k = \delta_{k3}$ in Eqs. (8.7) and (8.6) we obtain

$$j_{11} = \frac{J^0}{2}\left[1 - \cos\left(2\theta_0 + \frac{2\pi z}{p} - 2\phi\right)\right],$$

$$j_{22} = \frac{J^0}{2} 1 + \cos \; 2\theta_0 + \frac{2\pi z}{p} - 2\phi$$

$$j_{33} = J^0$$

$$j_{12} = \frac{J^0}{2}\sin\left(2\theta_0 + \frac{2\pi z}{p} - 2\phi\right), \qquad j_{13} = j_{23} = 0 \tag{8.11}$$

Equations (8.11) are the solution of the equations of microinertia conservation.

Maxwell's equations for the magneto-static case read

$$\nabla \times \mathbf{H} = \mathbf{0}, \qquad \nabla \cdot \mathbf{B} = 0 \tag{8.12}$$

Since $\mathbf{H}$ and $\mathbf{B}$ are functions of z only, we have the solutions

$$H_1 = \text{const.}, \qquad H_2 = \text{const.}, \qquad B_3 = \text{const.}$$

By taking

$$H_1 = 0, \qquad H_2 = H = \text{const.}, \qquad B_3 = 0 \tag{8.13}$$

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all boundary conditions $[H_1] = 0, [H_2] = 0$ and $[B_2] = 0$ are satisfied. The constitutive Eq. (5.7), with the consideration that $j_{13} = j_{23} = 0$ give

$$\begin{aligned} M_1 &= B_1 = (\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} + \mu_3 j_{11} + \bar{\mu}_2 \phi')B_1 + \mu_3 j_{12} B_2, \\ M_2 &= B_2 - H = \mu_3 j_{21} B_1 + (\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} + \mu_3 j_{22} + \bar{\mu}_2 \phi')B_2, \\ M_3 &= 0 \end{aligned} \tag{8.14}$$

From these we solve M_1 and M_2 in terms of H:

$$M_1 = \chi_1 H, \qquad M_2 = -(1 + \chi_2)H \tag{8.15}$$

where

$$\begin{aligned} \chi_1 &= \frac{\mu_3 j_{12}}{\Delta_0 + \Delta_1 \phi' + \Delta_2 \phi'^2}, \\ \chi_2 &= \frac{\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} + \mu_3 j_{11} - 1 + \bar{\mu}_2 \phi'}{\Delta_0 + \Delta_1 \phi' + \Delta_2 \phi'^2} \\ \Delta_0 &= (\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} - 1)^2 + \mu_3(\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} - 1)(j_{11} + j_{22}) \\ &\quad + \mu_3^2(j_{11} j_{22} - j_{12}^2), \\ \Delta_1 &= \bar{\mu}_2[2(\mu_1 + \mu_2 \operatorname{tr} \mathbf{j} - 1) + \mu_3(j_{11} + j_{22})], \\ \Delta_2 &= \bar{\mu}_2^2 \end{aligned} \tag{8.16}$$

Equations of the balance of momenta Eqs. (3.3) and (3.4) take the forms:

$$-p_{,k} - \rho(A^0_{33}\phi' + A^1_{33}\phi'^2 + A^2_{33}\phi'^3)\delta_{k3} + \tfrac{1}{2}(M^2)'\delta_{k3} = 0, \tag{8.17}$$

$$\rho(A^0_{3k} + A^1_{3k}\phi' + A^2_{3k}\phi'^2) + \rho\varepsilon_{kl3} (A^0_{l3}\phi' + A^1_{l3}\phi'^2 + A^2_{l3}\phi'^3) + M_1 H \delta_{k3} = 0 \tag{8.18}$$

An examination of these equations for $k = 1, 2$ show that Eq. (8.17) gives $p = p(z)$ and Eq. (8.18) is automatically satisfied. The remaining two equations (for $k = 3$) give

$$-p - \rho(A^0_{33}\phi' + A^1_{33}\phi'^2 + A^2_{33}\phi'^3) + \tfrac{1}{2}[\chi_1^2 + (1 + \chi_2)^2]H^2 = -p_0 \tag{8.19}$$

$$\rho(A^0_{33} + A^1_{33}\phi' + A^2_{33}\phi'^2)' + \chi_1 H^2 = 0 \tag{8.20}$$

where p_0 is a constant pressure. Upon solving Eq. (8.20) we obtain $\phi(z)$ and then Eq. (8.19) determines the pressure, p. Equation (8.20) is however highly nonlinear, and closed form solution appear to be out of our reach. We study two special cases:

i) *Second order effects are negligible* If the products and squares of γ and $\mathbf{B}$ are negligible then Eq. (8.20) can be integrated exactly. This amounts to

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neglecting the curvature piezomagnetism and squares of γ, i.e. $\bar{A}_i = \bar{\mu}_2 = 0$. In this case we have $A^0_{33} = A^2_{33} = \Delta_1 = \Delta_2 = 0$ and A^1_{33} and Δ_0 are constants so that Eq. (8.20) may be transformed to

$$\theta'' + \lambda^2 H^2 \sin\theta \cos\theta = 0 \tag{8.21}$$

where we set $\theta_0 = \pi/2$ by taking the y-axis along the long axes of the molecules at $z = 0$-plane, and wrote:

$$\theta = \phi - \frac{\pi z}{p}, \qquad \lambda = \left(\frac{\mu_3 J^0}{\rho \Delta_0 A^1_{33}}\right)^{1/2} \tag{8.22}$$

Equation (8.21) is identical to the one obtained by the energy method (cf. Ref. [3, p. 632]). The first integral of Eq. (8.21) is

$$\theta'^2 + \lambda^2 H^2 \sin^2\theta = \frac{C_1^2}{h^2} \tag{8.23}$$

where C_1^2 is a constant. Once more integration of Eq. (8.23) gives

$$\pm(z - z_0)\frac{C_1}{h} = \int^{\theta} (1 - \kappa^2 \sin^2\theta')^{-1/2}\, d\theta' \tag{8.24}$$

where

$$\kappa = \frac{\lambda h H}{C_1} \tag{8.25}$$

Because of the symmetry about $z = 0$-plane we have $\theta(-z) = -\theta(z)$ and $\theta(0) = 0$. Consequently Eq. (8.24) reduces to

$$\frac{C_1 z}{h} = \int_0^{\theta} (1 - \kappa^2 \sin^2\theta')^{-1/2}\, d\theta' = F(\kappa, \theta),$$

$$0 \le z \le h, \qquad 0 \le \theta \le \pi \tag{8.26}$$

where $F(\kappa, \theta)$ is the elliptic integral of the first kind. The constant C_1 is determined by the boundary condition at $z = h$.

$$C_1 = F(\kappa, \theta_h), \qquad \theta_h = \frac{\pi h}{p}, \qquad 0 \le \theta_h \le \pi \tag{8.27}$$

where we assumed that the cholesteric elements on the wall are fixed at an angle θ_h with the y-axis. The solution is therefore complete since in the region $-h \le z < 0$ it is given by $\theta(-z) = -\theta(z)$.

We may inquire: if a critical field H_c exist for which all cholesteric elements become parallel to each other, i.e. cholesteric liquid become nematic. In this case, of course, we must lift the boundary conditions at the walls. The answer to this is pursued in the same way as is done in the literature. We therefore do not pursue this problem further.

ii) *Effect of curvature piezo-magnetism* We again set $\bar{A}_i = 0$ but keep $\bar{\mu}_2$. As expected $\bar{\mu}_2$ is probably a very small quantity in the sense that

$$\left| \frac{\bar{\mu}_2}{p\mu_1} \right| \ll 1 \tag{8.28}$$

If this is the case, we may use $\bar{\mu}_2$ as a perturbation parameter. Equation (8.20), to the first degree in $\bar{\mu}_2$, takes the form

$$\theta'' + \lambda^2 H^2 \left(1 + \frac{\pi\Delta_1}{p\Delta_0}\right) \sin\theta \cos\theta = 0 \tag{8.29}$$

It is interesting that this equation is of the same form as Eq. (8.21) with the only change that a factor $1 + \pi\Delta_1/p\Delta_0$ multiplies λ^2. Consequently by redefining κ as

$$\kappa = \lambda \left(1 + \frac{\pi\Delta_1}{p\Delta_0}\right)^{1/2} \frac{hH}{C_1} \tag{8.30}$$

the previous solution remains valid in this case also. Physically this change implies a spatial period corresponding to the variation of θ by 2π in the amount

$$T = \left(\frac{4h}{C_1}\right) K(\kappa) \tag{8.31}$$

with κ is now given by Eq. (8.30). Here $K(\kappa)$ is the complete elliptic integral.

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